K10. Land surface observation and assimilation: data-model integration approach for improved hydrologic prediction and water resources management

Combining Geophysical Variables for Maximizing Temporal Correlation Without Reference Data

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Motivations

- In many applications using remote sensing data, the **temporal correlation** is importantly used

INT. J. REMOTE SENSING, 2002, VOL. 23, NO. 18, 3873–3878

Taylor & Francis
Taylor & Francis Group

Global correlation analysis for NDVI and climatic variables and NDVI trends: 1982–1990

IEEE GEOSCIENCE AND REMOTE SENSING LETTERS, VOL. 9, NO. 4, JULY 2012

Microwave Satellite Data for Hydrologic Modeling in Ungauged Basins

Sadiq I. Khan, Yang Hong, Humberto J. Vergara, Jonathan J. Gourley, G. Robert Brakenridge, Tom De Groeve, Zachary L. Flamig, Frederick Policelli, and Bin Yong

- **Bias correction and Scaling** (e.g. CDF matching, linear scaling) against a reference can be selectively applied using a reliable reference

Linear combination for maximizing correlation

$$\boldsymbol{\theta}_{\boldsymbol{c}} = w\boldsymbol{\theta}_1 + (1 - w)\boldsymbol{\theta}_2$$

$$\text{Maximize } \rho_{c,t} = f(w) = \frac{E[(\boldsymbol{\theta}_{\boldsymbol{c}} - \mu_c)(\boldsymbol{\theta}_{\boldsymbol{t}} - \mu_t)]}{\sigma_c \sigma_t}$$

Subject to $0 \leq w \leq 1$

Linear combination for maximizing correlation

Where,

- $\rho_{c,t} = \frac{Cov(\theta_1,t)w + Cov(\theta_2,t)(1-w)}{\sigma_c \sigma_t}$

- ${\sigma_c}^2 = {\sigma_1}^2 w^2 + 2Cov(\theta_1, \theta_2)w(1-w) + {\sigma_2}^2(1-w)^2$

- $\mu_c = w\mu_1 + (1-w)\mu_2$

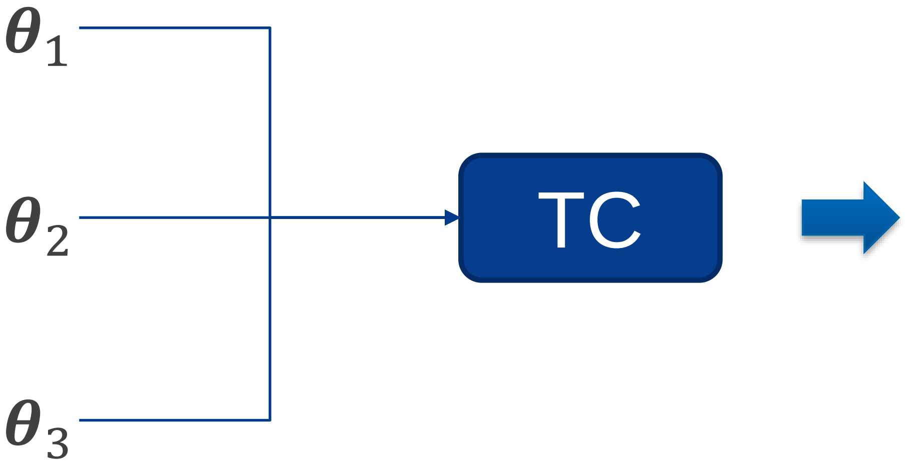
Linear combination for maximizing correlation

$$\frac{\delta \rho_{c,t}}{\delta w} = 0$$

$$w = \frac{\sigma_2(\rho_{1t} - \rho_{12} \cdot \rho_{2t})}{\sigma_1(\rho_{2t} - \rho_{12} \cdot \rho_{1t}) + \sigma_2(\rho_{1t} - \rho_{12} \cdot \rho_{2t})}$$

Kim et al.(2015)

Triple Collocation (TC)



$$\sigma_\varepsilon = \begin{bmatrix} \sqrt{Q_{11} - \frac{Q_{12}Q_{13}}{Q_{23}}} \\ \sqrt{Q_{22} - \frac{Q_{12}Q_{23}}{Q_{13}}} \\ \sqrt{Q_{33} - \frac{Q_{13}Q_{23}}{Q_{12}}} \end{bmatrix}$$

$$\rho_{t,X} = \pm \begin{bmatrix} \sqrt{\frac{Q_{12}Q_{13}}{Q_{11}Q_{23}}} \\ \mathrm{sign}(Q_{13}Q_{23})\sqrt{\frac{Q_{12}Q_{23}}{Q_{22}Q_{13}}} \\ \mathrm{sign}(Q_{12}Q_{23})\sqrt{\frac{Q_{13}Q_{23}}{Q_{33}Q_{12}}} \end{bmatrix}$$

Stoffelen (1998); McColl et al.(2015)

Four TC assumptions

- Linearity between observations and the truth: $\boldsymbol{\theta_i} = \alpha_i \boldsymbol{t} + \beta_i + \boldsymbol{\varepsilon_i}$

How do the assumptions affect the combination?

- Zero error-cross correlation (ECC): $\rho_{\varepsilon_i,\varepsilon_j} = 0$

- Error-truth orthogonality: $\rho_{\varepsilon_i,t} = 0$

Experiments using synthetic data

$$t \sim U(0,1)$$

$$\boldsymbol{\theta_1} = \boldsymbol{t} + \boldsymbol{\varepsilon_1}$$
$$\boldsymbol{\theta_2} = \boldsymbol{t} + \boldsymbol{\varepsilon_2}$$
$$\boldsymbol{\theta_3} = \boldsymbol{t} + \boldsymbol{\varepsilon_3}$$

TC

$$\rho_{1,t}, \rho_{2,t}$$

$$w = \frac{\sigma_2\left(\rho_{1,t} - \rho_{1,2} \cdot \rho_{2,t}\right)}{\sigma_1\left(\rho_{2,t} - \rho_{12} \cdot \rho_{1t}\right) + \sigma_2\left(\rho_{1,t} - \rho_{1,2} \cdot \rho_{2,t}\right)}$$

$$\boldsymbol{\theta_c} = w\boldsymbol{\theta_1} + (1 - w)\boldsymbol{\theta_2}$$

Experiments using synthetic data

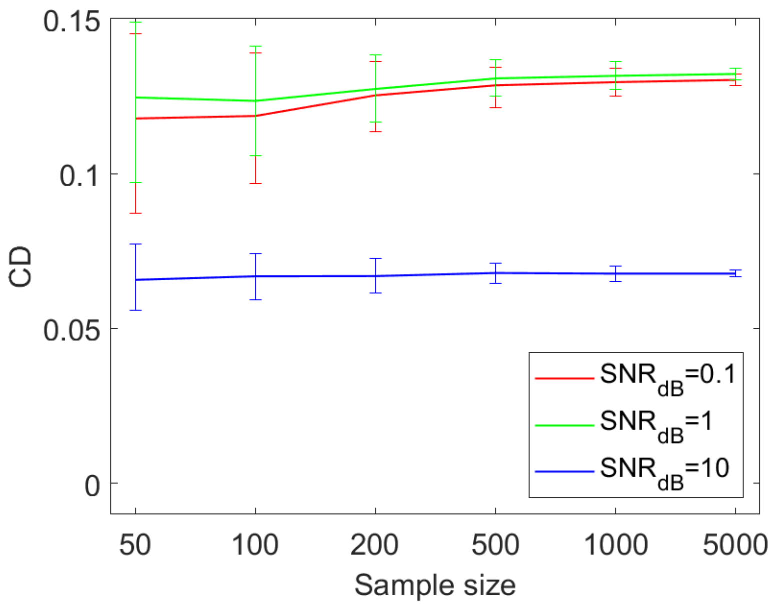
- **Exp 1**: sample sizes
- **Exp 2**: error stationary
- **Exp 3**: error cross-correlation
- **Exp 4**: error-truth orthogonality

✓ Correlation Difference (**CD**) $= \rho_{C,t} - \rho_{parent,t}$

✓ $\mathrm{SNR} = 10^{\frac{\mathrm{SNR_{dB}}}{10}} = \frac{\mathrm{P_S}}{\mathrm{P_N}} = \frac{E[S^2]}{\sigma_\mathrm{N}^2}$ ➡ $\mathrm{SNR_{dB}} = [0.1, 1, 10]$

Exp 1: sample size

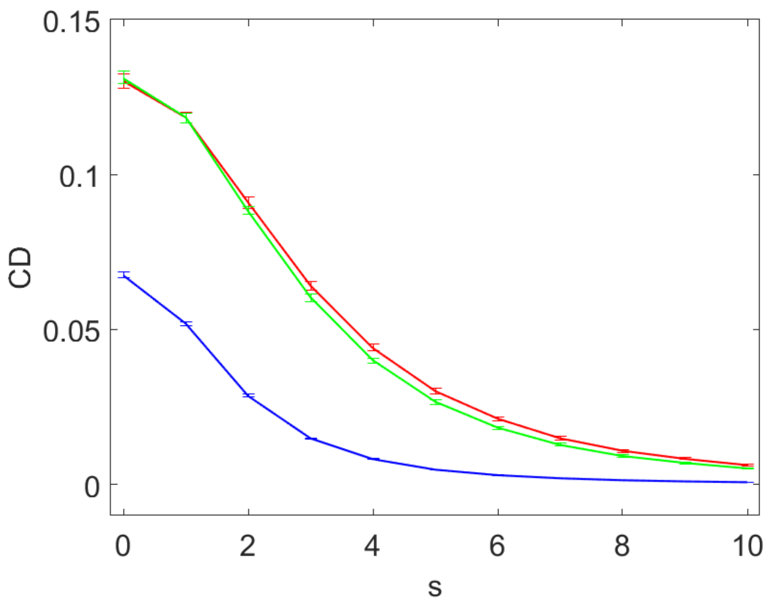
- $\varepsilon_i = \mathrm{N}(0, \sqrt{\mathrm{P_N}})$
- sample sizes (L)=[50, 100, 500, 1000, 5000]



Exp 2: error stationarity

- $\boldsymbol{\varepsilon_i} = \boldsymbol{a} + \frac{\mathrm{H}-0.5\mathrm{L}}{\mathrm{L}} \cdot \mathrm{s} \cdot \mathrm{E}[\boldsymbol{t}]$

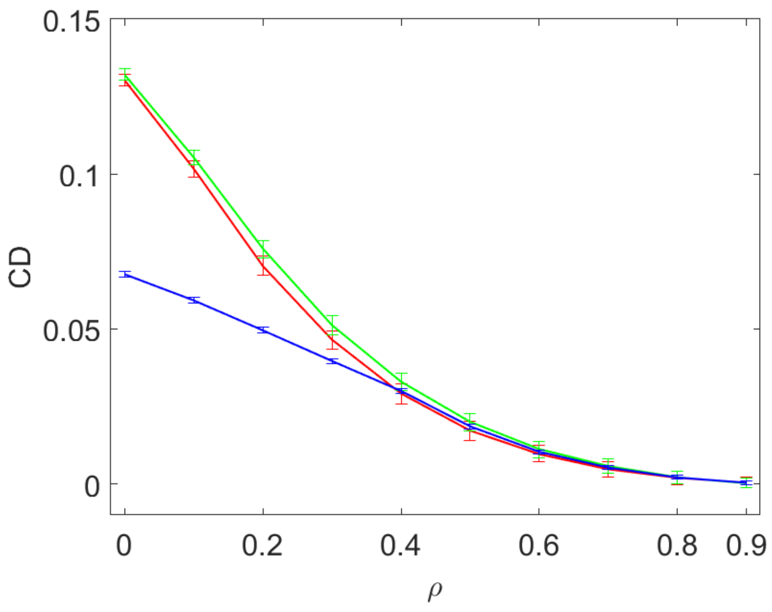
    - $\mathrm{H} = 1{:}\,\mathrm{L}$
    - s= 0:1:10



Exp 3: error cross-correlation

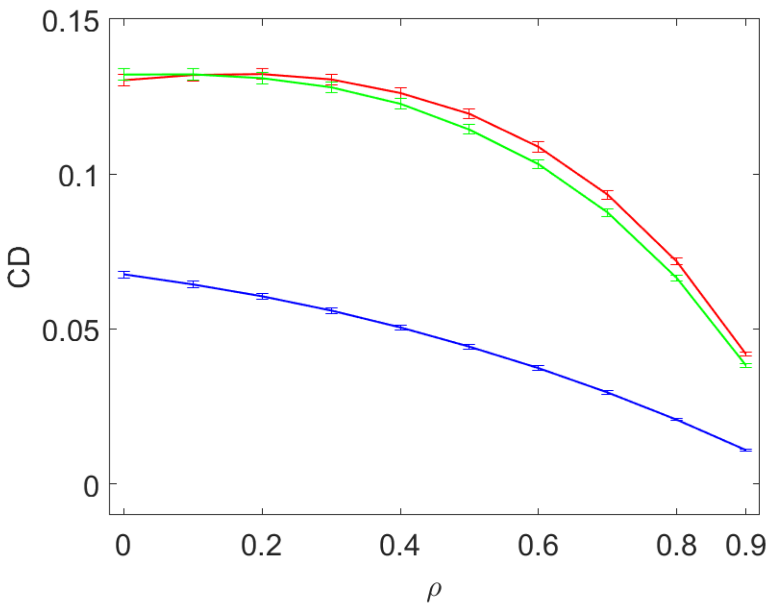
- $\boldsymbol{\varepsilon_1} = \mathrm{N}\left(0, \sqrt{\mathrm{P_N}}\right)$
- $\boldsymbol{\varepsilon_2} = \rho \times \boldsymbol{\varepsilon_1} + \sqrt{1-\rho^2} \times \mathrm{N}\left(0, \sqrt{\mathrm{P_N}}\right)$
- $\boldsymbol{\varepsilon_3} = \rho \times \boldsymbol{\varepsilon_2} + \sqrt{1-\rho^2} \times \mathrm{N}\left(0, \sqrt{\mathrm{P_N}}\right)$

  - $\rho = 0.0{:}0.1{:}0.9$



Exp 4: error orthogonality

- $\boldsymbol{\varepsilon_i} = \rho \times \boldsymbol{t}/\sqrt{SNR} + \sqrt{1-\rho^2} \times \mathrm{N}\left(0, \sqrt{\mathrm{P_N}}\right)$
  - $\rho = 0.0{:}0.1{:}0.9$



Experiments using soil moisture data

No	Product	Microwave Band	Type
①	SMOS	L	Passive
②	ASCAT	C	Active
③	SMAP L3	L	Passive
④	MERRA2		Reanalysis

Combination Method
1 (replacing) + 2 (fixed) scheme (1P2)

$\boldsymbol{\theta}_1 \quad \boldsymbol{\theta}_2 \quad \boldsymbol{\theta}_3 \quad \boldsymbol{\theta}_4$

TC

$\boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4 \rightarrow \boldsymbol{\rho}_{1,t}$

$$w = \frac{\sigma_2\left(\rho_{1,t} - \rho_{1,2} \cdot \rho_{2,t}\right)}{\sigma_1\left(\rho_{2,t} - \rho_{1,2} \cdot \rho_{1,t}\right) + \sigma_2\left(\rho_{1,t} - \rho_{1,2} \cdot \rho_{2,t}\right)}$$

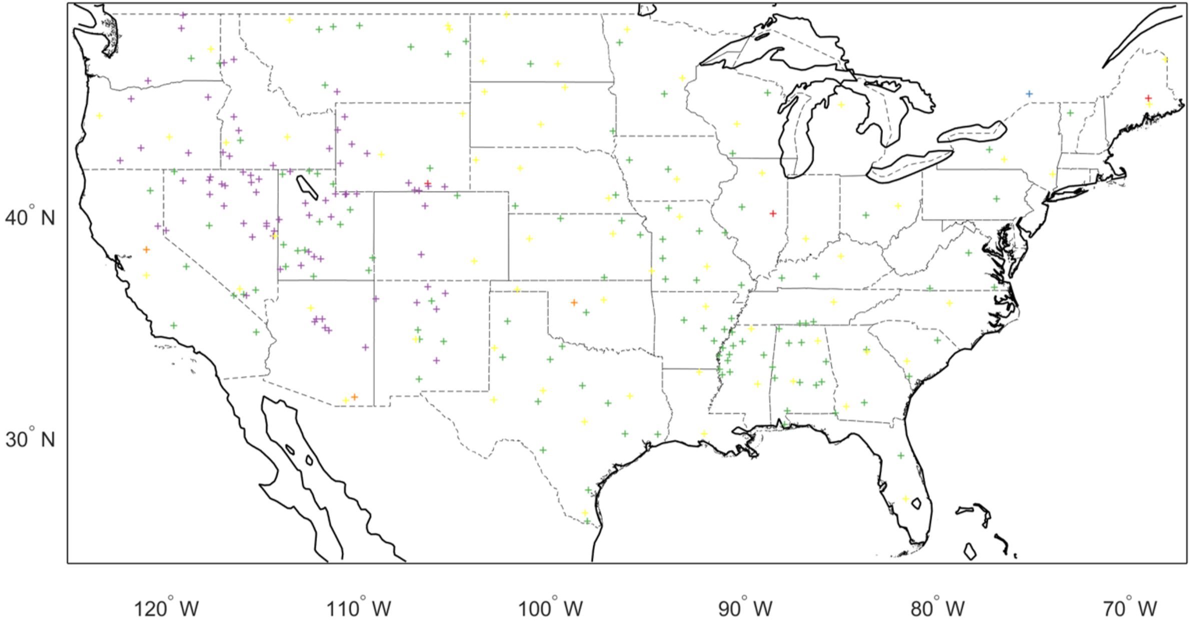
TC

$\boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4 \rightarrow \boldsymbol{\rho}_{2,t}$

Six combination cases

Case	1 (replacing)	2 (fixed)
(1)	① or ②	③, ④
(2)	① or ③	②, ④
(3)	① or ④	②, ③
(4)	② or ③	①, ④
(5)	② or ④	①, ③
(6)	③ or ④	①, ②

Ground stations for validation



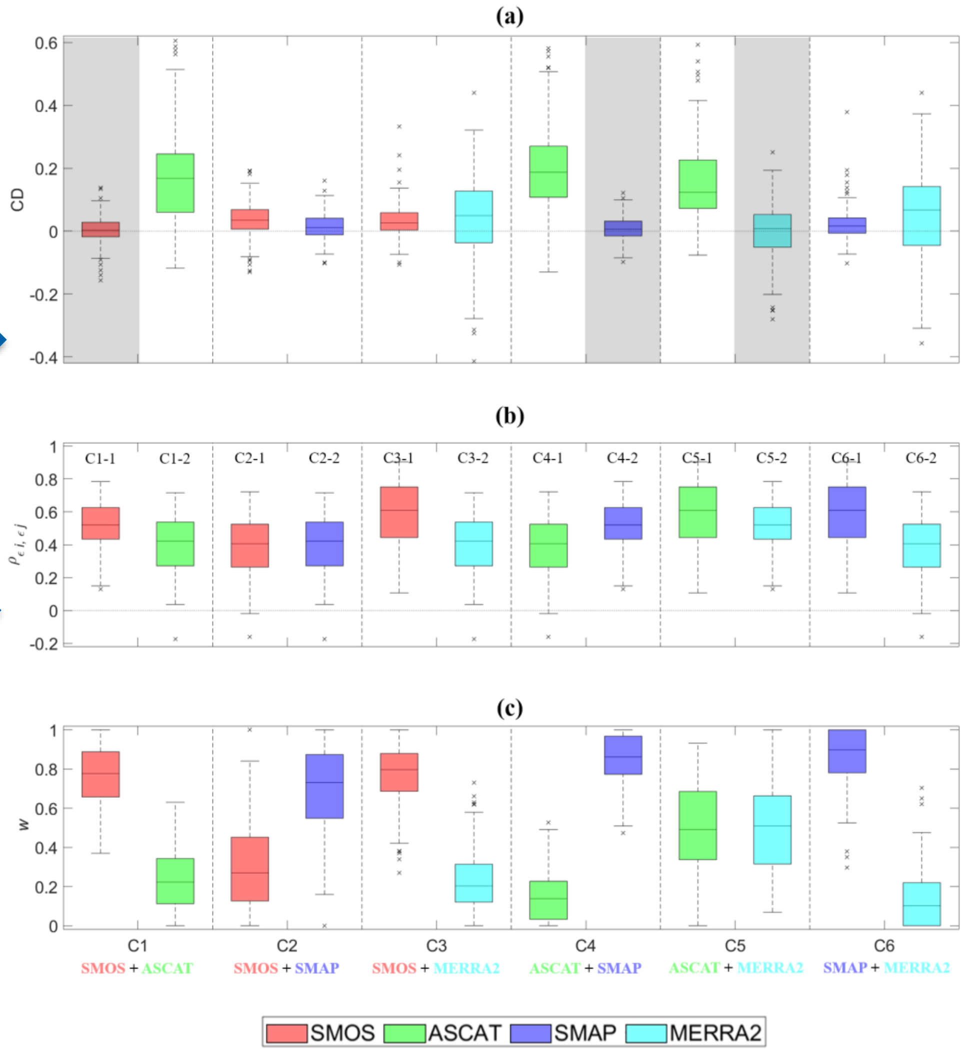
Network	#Stations
COSMOS	3
RISMA	1
SCAN	121
SNOTEL	77
SOILSCAPE	3
USCRN	66
Total	**271**

SM Combination results

9/12 of E[CD] > 0

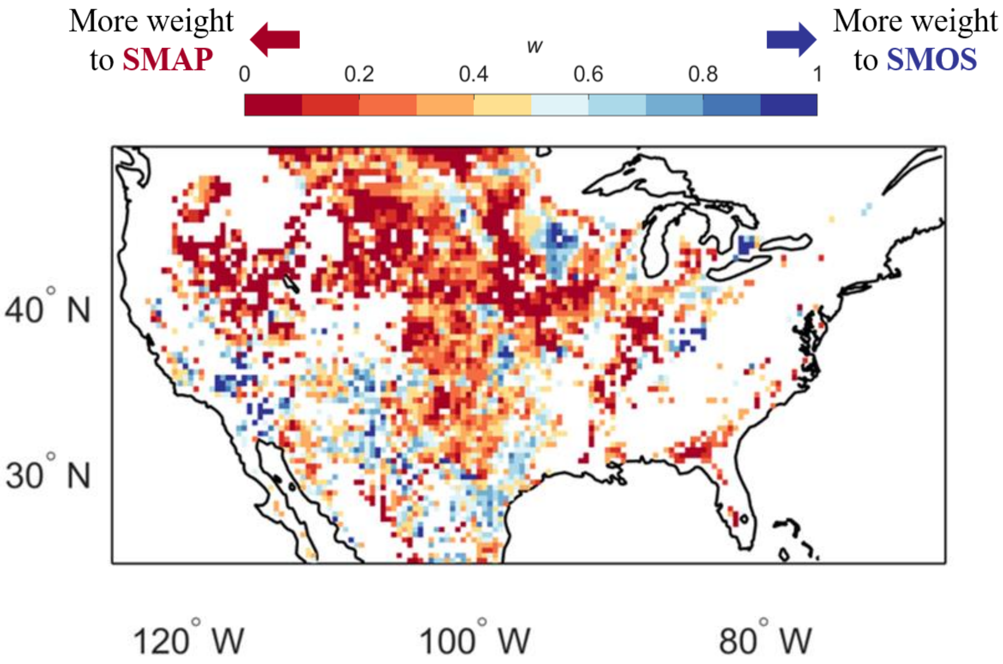
$\rho_{\varepsilon_i,\varepsilon_j} \neq 0$

Varied *w*



water@ UNSW
water research centre

Spatial distribution of *w* (SMAP+SMOS)



Max. R vs. Min. MSE

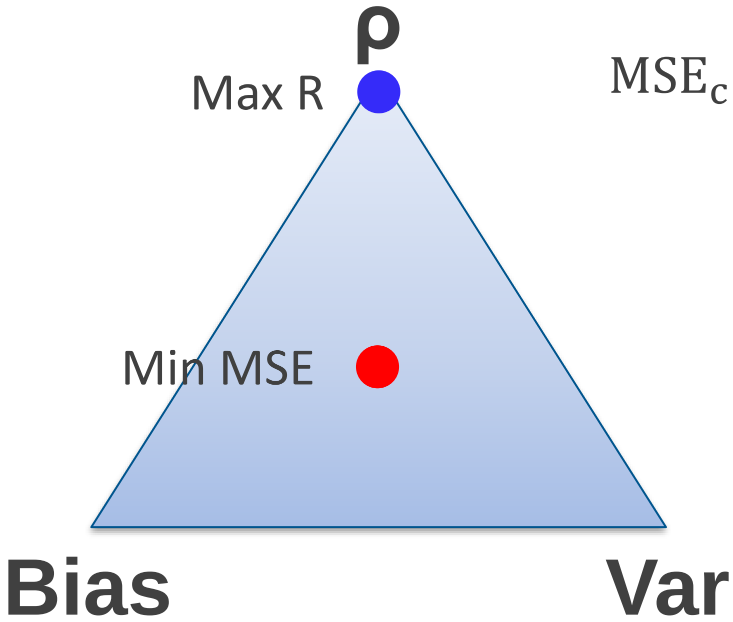
$$w_{\mathrm{maxR}} = \frac{\sigma_2\left(\rho_{1,t} - \rho_{1,2}\rho_{2,t}\right)}{\sigma_1\left(\rho_{2,t} - \rho_{1,2}\rho_{1,t}\right) + \sigma_2\left(\rho_{1,t} - \rho_{1,2}\rho_{2,t}\right)}$$

$$\Updownarrow$$

$$\mathrm{w}_{min\mathrm{MSE}} = \frac{{\sigma_2}^2 - Cov(\theta_1, \theta_2) + Cov(\theta_1, \theta_t) - Cov(\theta_2, \theta_t)}{{\sigma_1}^2 + {\sigma_2}^2 - 2Cov(\theta_1, \theta_2)}$$

Max. R vs. Min. MSE

Gupta et al. (2009)



$$\mathrm{MSE_c} = 2 \cdot \sigma_t \cdot \sigma_c \cdot \left(1 - \rho_{c,t}\right) + (\mu_t - \mu_c)^2 + (\sigma_t - \sigma_c)^2$$

Max R →

- Bias correction
- Scaling

Conclusions

- **Complementarity** exists among various products
- The **TC-based linear combination** is a simple but effective way to take the complementarity
- **Sample size** should be long enough (>500) to properly obtain weights
- **ECC** should be carefully considered in the process
- **Future works:** comparisons of max R and min MSE, extension to three or more products, estimating ECC